# SUPPLEMENTAL MATERIALS

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# Flood Risk Mitigation and Valve Control in Stormwater Systems: State-Space Modeling, Control Algorithms, and Case Studies

Marcus N. Gomes Jr., Marcio H. Giacomoni, Ahmad F. Taha, and Eduardo M. Mendiondo

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The following table shows all variables used in the model. We categorize the variables into (i) system (i.e., coupled watershed, reservoir, and channel), (ii) watershed, (iii) reservoir, and (iv) channel. The following table has the variable description for each category with units in MLT units, where $n$ is the number of states, $q$ is the number of cells in the watershed, $n_r$ is the number of reservoirs per watershed, $n_c$ is the number of sub-reaches, and $s$ is the number of systems composed of a watershed, $n_r$ reservoirs per watershed, and a channel. Moreover, superscripts $w$, $r$, and $c$ represent watersheds, reservoirs, and channels, respectively, whereas subscript $i$ represents a specific sub-reach in the channel. The dimension N.A means not applicable.

**Table 1.** Variables used in the Hydrologic-Hydraulic Model

| Category | Variable | Definition | Dimension | Unit |
|---|---|---|---|---|
| System | $\boldsymbol{E}$ | Singular matrix for DAE dynamics | $\mathbb{R}^{n\times n}$ | N.A |
| | $\boldsymbol{A}$ | System matrix | $\mathbb{R}^{n\times n}$ | N.A |
| | $\boldsymbol{B}$ | Control matrix | $\mathbb{R}^{n\times n}$ | N.A |
| | $\boldsymbol{\psi}$ | Disturbance vector | $\mathbb{R}^{n\times n}$ | N.A |
| | $\boldsymbol{C}$ | Output matrix | $\mathbb{R}^{n\times n}$ | L |
| | $\Delta t$ | Time-step in time units | $\mathbb{R}$ | T |
| Watershed | $\boldsymbol{h}_{ef}^{w}$ | Water depth in watershed cells | $\mathbb{R}^{q}$ | L |
| | $\boldsymbol{f}_{d}$ | Cumulative infiltration depth | $\mathbb{R}^{q}$ | L |
| | $q_{out}^{w}$ | Watershed outflow | $\mathbb{R}$ | $L^3T^{-1}$ |
| | $\boldsymbol{c}$ | Infiltration capacity | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{k}_{sat}$ | Saturated hydraulic conductivity | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{\zeta}$ | Suction head | $\mathbb{R}^{q}$ | L |
| | $\boldsymbol{\theta}_s - \boldsymbol{\theta}_i$ | Effective soil moisture | $\mathbb{R}^{q}$ | N.A |
| | $\boldsymbol{q}_{in}$ | Boundary inflows in watershed cells | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{i}_{p}$ | Rainfall intensity | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{e}_{TR}$ | Evapotranspiration | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{f}$ | Infiltration rate | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\boldsymbol{s}$ | Surface runoff storage in each cell | $\mathbb{R}^{q}$ | $L^3$ |
| | $\omega^{w}$ | Cell area in watersheds | $\mathbb{R}$ | $L^2$ |
| | $\boldsymbol{q}_{out}$ | Boundary outflows in watershed cells | $\mathbb{R}^{q}$ | $LT^{-1}$ |
| | $\Delta x^{w}$ | Length of cells in watersheds | $\mathbb{R}$ | L |
| | $\Delta y^{w}$ | Width of cells in watersheds | $\mathbb{R}$ | L |
| | $\boldsymbol{h}_{0}$ | Initial abstraction | $\mathbb{R}^{q}$ | L |
| | $\boldsymbol{s}_{0}$ | Bottom slope in watersheds | $\mathbb{R}^{q}$ | $LL^{-1}$ |
| | $\boldsymbol{n}$ | Manning's roughness coefficient in watersheds | $\mathbb{R}^{q}$ | $TL^{-1/3}$ |
| | $\boldsymbol{\lambda}$ | Overland flow parameter for watershed cells | $\mathbb{R}^{q}$ | $(LT^{-1})^{3/5}$ |
| | $\boldsymbol{B}_{d}$ | Direction matrix | $\mathbb{R}^{q\times q}$ | N.A |
| | $\boldsymbol{\psi}^{w}$ | Watershed disturbance vector | $\mathbb{R}^{2q+1}$ | N.A |
| Reservoir | $q_{o}$ | Orifice outflow | $\mathbb{R}$ | $L^3T^{-1}$ |
| | $q_{s}$ | Spillway outflow | $\mathbb{R}$ | $L^3T^{-1}$ |
| | $a_{o}$ | Orifice area | $\mathbb{R}$ | $L^2$ |
| | $k_{o}$ | Orifice coefficient | $\mathbb{R}$ | $L^{5/2}T^{-1}$ |
| | $\hat{h}^{r}$ | Effective orifice pressure | $\mathbb{R}$ | L |
| | $h_{o}$ | Orifice bottom depth | $\mathbb{R}$ | L |
| | $l_{ef}$ | Effective spillway length | $\mathbb{R}$ | L |
| | $p$ | Spillway elevation from reservoir bottom | $\mathbb{R}$ | L |
| | $k_{s}$ | Spillway coefficient | $\mathbb{R}$ | $L^{3/2}T^{-1}$ |
| | $c_{d,s}$ | Spillway discharge coefficient | $\mathbb{R}$ | N.A |
| | $\omega^{r}$ | Reservoir area | $\mathbb{R}$ | $L^2$ |
| | $s^{r}$ | Reservoir storage | $\mathbb{R}$ | $L^3$ |
| | $\eta$ | Reservoir porosity | $\mathbb{R}$ | N.A |
| | $\boldsymbol{\varphi}_{out,s}^{r}$ | Concatenated reservoir outflow | $\mathbb{R}^{n_r s}$ | $L^3T^{-1}$ |
| | $\boldsymbol{y}_{s}^{r}$ | Concatenated reservoir water depths | $\mathbb{R}^{n_r s}$ | L |
| | $\boldsymbol{\sigma}_{s}^{r}$ | Concatenated reservoir control signals | $\mathbb{R}^{n_r s}$ | N.A |
| | $\boldsymbol{q}_{out}^{r}$ | Reservoir outflow | $\mathbb{R}^{n_r s}$ | $L^3T^{-1}$ |
| | $\boldsymbol{u}$ | Control signal | $\mathbb{R}^{n_r}$ | N.A |
| | $\boldsymbol{h}^{r}$ | Water depth in reservoirs | $\mathbb{R}^{n_r}$ | L |
| Channel | $\boldsymbol{q}_{out}^{c}$ | Channel flow in all sub-reaches | $\mathbb{R}^{n_c}$ | $L^3T^{-1}$ |
| | $a_{i}$ | Wetted area in channel in sub-reach i | $\mathbb{R}$ | $L^2$ |
| | $r_{h,i}$ | Hydraulic radius in sub-reach i | $\mathbb{R}$ | L |
| | $n_{i}$ | Manning's roughness coefficient in sub-reach i | $\mathbb{R}$ | $TL^{-1/3}$ |
| | $\boldsymbol{a}^{c}$ | Wetted area in all sub-reaches | $\mathbb{R}^{n_c}$ | $L^2$ |
| | $s_{f,i}$ | Friction slope for sub-reach i | $\mathbb{R}$ | $LL^{-1}$ |
| | $\boldsymbol{h}^{c}$ | Water depths in channel sub-reaches | $\mathbb{R}^{n_c}$ | L |
| | $\Delta x$ | Sub-reach width | $\mathbb{R}^{n_c}$ | L |
| | $\Delta y$ | Sub-reach length | $\mathbb{R}^{n_c}$ | L |
| | $\boldsymbol{q}_{in}^{c}$ | Inflow in sub-reaches | $\mathbb{R}^{n_c}$ | $L^3.T^{-1}$ |
| | $e_{l,i}$ | Central elevation of sub-reach i | $\mathbb{R}$ | L |
| | $\boldsymbol{A}_{slope}$ | Boundary relationship between sub-reach cells | $\mathbb{R}^{n_c\times n_c}$ | N.A |
| | $\boldsymbol{b}_{slope}$ | Vector containing outlet boundary conditions | $\mathbb{R}^{n_c}$ | N.A |
| | $\boldsymbol{B}_{d}^{c}$ | Direction matrix for channel sub-reaches | $\mathbb{R}^{n_c\times n_c}$ | N.A |
| | $\boldsymbol{w}$ | Vector containing inlet and outlet boundary conditions | $\mathbb{R}^{n_c}$ | N.A |